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Title:

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A Modified BFGS Update for Microwave Circuit Design using Aggressive Space Mapping

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***Abstract*—We present a novel rank-2 update formula for nonlinear optimization problems. The currently available BFGS rank-2 formula is primarily designed to update symmetric positive definite Hessian matrices. Our proposed modified rank-2 BFGS formula updates non-symmetric rectangular Jacobian matrices. The novel updating formula is illustrated through applying the aggressive space mapping algorithm to two microwave circuits. Our new updating formula shows a convergence rate similar to or faster than the available rank-1 Broyden update.**

***Index Terms*—BFGS update, Broyden update, CAD, EM optimization, EM simulation, filter design, space mapping.**

I. Introduction

The space mapping (SM) approach aims at optimizing a fine electromagnetic (EM) model using a much faster but less accurate coarse model. Aggressive space mapping (ASM) iteratively updates the mapping $\boldsymbol{P}$ between the parameter spaces of a coarse and a fine model [1]. The mapping Jacobian is approximated by a matrix $\boldsymbol{B}$, i.e., $\boldsymbol{B} \approx \boldsymbol{J}_P(\boldsymbol{x}_f)$. Different schemes proposed in the literature to update the matrix $\boldsymbol{B}$ are reviewed in [1]. In the aggressive SM approach [2], a proposed technique based on the Broyden rank-1 formula [3] is employed to update $\boldsymbol{B}$. The Broyden-based scheme exhibits good results [2]. Rank-1 updates give limited degrees of freedom in changing the updated matrix components. This may slow the convergence to the optimal design. Only Broyden rank-1 formula is currently available for updating the non symmetric Jacobian matrices in solving nonlinear systems.

We propose a novel modified BFGS-type rank-2 updating formula for the non-symmetric case, e.g., the Jacobian, which is in general a rectangular matrix. The proposed formula is compared with the Broyden formula using two examples. It provides the same or better convergence results for the considered examples using the aggressive SM algorithm.

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II. Theoretical Background

The aggressive SM solves the nonlinear system [2]

$$\boldsymbol{f} \triangleq \boldsymbol{f}(\boldsymbol{x}_f) = \boldsymbol{P}(\boldsymbol{x}_f) - \boldsymbol{x}_c^* = \boldsymbol{0} \tag{1}$$

for $\boldsymbol{x}_f \in X_f \subseteq \mathbb{R}^n$, where $\boldsymbol{x}_c^*$ is the optimal coarse model solution. Here we use the vector valued function $\boldsymbol{f}: \mathbb{R}^n \mapsto \mathbb{R}^m$ to represent a mapping between the coarse and fine models design spaces with different dimensionalities. According to the Newton method for nonlinear equations [4], the solution of (1) at the *j*th iteration is given by

$$\boldsymbol{x}_f^{(j+1)} = \boldsymbol{x}_f^{(j)} + \boldsymbol{h}^{(j)}, \quad \boldsymbol{J}_P^{(j)}\boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}. \tag{2}$$

A. The Broyden Method

Since the first-order information to evaluate $\boldsymbol{J}_P$ may be difficult to obtain, Broyden [3] suggested a formula which updates an estimate of the Jacobian matrix $\boldsymbol{B}^{(j+1)} \approx \boldsymbol{J}_P^{(j+1)}$ iteratively by satisfying the secant condition [4]

$$\boldsymbol{y}^{(j)} = \boldsymbol{B}^{(j+1)}\boldsymbol{h}^{(j)}, \tag{3}$$

where $\boldsymbol{B}^{(j+1)} \in \mathbb{R}^{m\times n}$, $\boldsymbol{h}^{(j)}$ and $\boldsymbol{y}^{(j)}$ are the difference between the successive iterates and the successive function values, respectively, i.e.,

$$\boldsymbol{h}^{(j)} = \boldsymbol{x}_f^{(j+1)} - \boldsymbol{x}_f^{(j)}, \qquad \boldsymbol{y}^{(j)} = \boldsymbol{f}^{(j+1)} - \boldsymbol{f}^{(j)}. \tag{4}$$

A correction matrix $\boldsymbol{C}^{(j)}$ is used to iteratively approximate the Jacobian matrix as [5]

$$\boldsymbol{B}^{(j+1)} = \boldsymbol{B}^{(j)} + \boldsymbol{C}^{(j)}. \tag{5}$$

In the case of a rank-1 updating matrix, the Broyden non-symmetric rank-1 formula for updating Jacobians is [3], [4]

$$\boldsymbol{B}^{(j+1)} = \boldsymbol{B}^{(j)} + \frac{\boldsymbol{y}^{(j)} - \boldsymbol{B}^{(j)}\boldsymbol{h}^{(j)}}{\boldsymbol{h}^{(j)T}\boldsymbol{h}^{(j)}}\boldsymbol{h}^{(j)T}. \tag{6}$$

B. The BFGS Formulation

The BFGS method was introduced in the context of quasi-Newton methods for nonlinear optimization [4] to update approximations to the Hessian matrix. The symmetric and positive definite Hessian approximating matrix accounts for the curvature measured during the most recent steps and it satisfies the secant condition [4]. Applying the BFGS updating formula to update the Jacobian matrix is not valid. This is because the conditions imposed to produce the BFGS formula, symmetry and positive definiteness of the Hessian matrix, do not hold in the case of the Jacobian matrix.

We propose a new rank-2 updating formula to update the Jacobian matrix used in solving the system of nonlinear equations (1). Here, we develop a non-symmetric rank-2 updating formula to adopt the Jacobian matrix characteristics. The proposed formula is based on the BFGS formulation.

C. A Non-Symmetric BFGS Updating Formula

In the case of a rank-2 updating matrix, the successive approximation formula (5) can be given by [5]

$$\boldsymbol{B}^{(j+1)} = \boldsymbol{B}^{(j)} + \alpha \boldsymbol{a}^{(j)} \boldsymbol{b}^{(j)T} + \beta \boldsymbol{c}^{(j)} \boldsymbol{d}^{(j)T}, \tag{7}$$

where α and $\beta \in \mathbb{R}$, $\boldsymbol{a}^{(j)}, \boldsymbol{c}^{(j)} \in \mathbb{R}^m$ and $\boldsymbol{b}^{(j)}$ and $\boldsymbol{d}^{(j)} \in \mathbb{R}^n$.

Here, for our proposed non-symmetric update, we choose

$$\begin{aligned} &\boldsymbol{a}^{(j)} = \boldsymbol{y}^{(j)},\ \boldsymbol{b}^{(j)} = \boldsymbol{h}^{(j)},\ \text{and} \\ &\boldsymbol{c}^{(j)} = \boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)},\ \boldsymbol{d}^{(j)} = \boldsymbol{B}^{(j)T} \boldsymbol{y}^{(j)}. \end{aligned} \tag{8}$$

Hence, the updating formula for the non-symmetric case becomes

$$\boldsymbol{B}^{(j+1)} = \boldsymbol{B}^{(j)} + \alpha \boldsymbol{y}^{(j)} \boldsymbol{h}^{(j)T} + \beta \left(\boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)}\right)\left(\boldsymbol{y}^{(j)T} \boldsymbol{B}^{(j)}\right). \tag{9}$$

We apply the secant condition (3) by multiplying both sides of (9) by $\boldsymbol{h}^{(j)}$ defined in (4):

$$\begin{aligned} \boldsymbol{y}^{(j)} &= \boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)} + \\ &\alpha \boldsymbol{y}^{(j)} \left(\boldsymbol{h}^{(j)T} \boldsymbol{h}^{(j)}\right) + \beta \left(\boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)}\right)\left(\boldsymbol{y}^{(j)T} \boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)}\right) \end{aligned} \tag{10}$$

We calculate the coefficients α and β to satisfy the secant condition as

$$\alpha = \frac{1}{\boldsymbol{h}^{(j)T} \boldsymbol{h}^{(j)}}, \quad \beta = \frac{-1}{\boldsymbol{y}^{(j)T} \boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)}}. \tag{11}$$

By substituting the values of α and β from (11) into (9), the proposed non-symmetric rank-2 updating formula for $\boldsymbol{B}^{(j+1)} \in \mathbb{R}^{m \times n}$ becomes

$$\boldsymbol{B}^{(j+1)} = \boldsymbol{B}^{(j)} + \frac{\boldsymbol{y}^{(j)} \boldsymbol{h}^{(j)T}}{\boldsymbol{h}^{(j)T} \boldsymbol{h}^{(j)}} - \frac{\boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)} \boldsymbol{y}^{(j)T} \boldsymbol{B}^{(j)}}{\boldsymbol{y}^{(j)T} \boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)}}. \tag{12}$$

III. Examples

We apply the aggressive SM algorithm to a seven-section transmission line impedance transformer and a six-section H-plane waveguide filter by solving (1). We compare the usage of the Broyden (6) and the proposed non-symmetric BFGS (12) updating formulas. In both examples, we use the least-squares Levenberg-Marquardt algorithm available in Matlab for the parameter extraction (PE) step [1]. The coarse model optimization uses the gradient-based minimax optimization routine presented in [6].

A. Seven-section Capacitively Loaded Impedance Transformer

The seven-section transmission line (TL) capacitively loaded impedance transformer example is described in [7]. We consider a “coarse” model as an ideal seven-section TL, where the “fine” model is a capacitively-loaded TL with capacitors $C_1 = \cdots = C_8 = 0.025$ pF. Design parameters are the normalized lengths $\boldsymbol{x}_f = \left[L_1\ L_2\ L_3\ L_4\ L_5\ L_6\ L_7\right]^T$, w.r.t. the quarter-wave length at 4.35 GHz. Design specifications are given by

$$\left|S_{11}\right| \le 0.07, \text{ for } 1 \text{ GHz} \le \omega \le 7.7 \text{ GHz} \tag{13}$$

with 68 points per frequency sweep. The characteristic impedances for the transformer are fixed as in [7]. We apply the ASM algorithm [2] utilizing the two formulas to update the mapping Jacobian matrix $\boldsymbol{B}$ utilizing 6 iterations.

Convergence results are given in Table I. The modified BFGS update provides better convergence w.r.t. the Broyden update. The reductions of $\|\boldsymbol{f}\|_2$ versus iteration using the Broyden and the modified BFGS formulas are shown in Fig 1. Fig. 2 shows the final response using the modified BFGS.

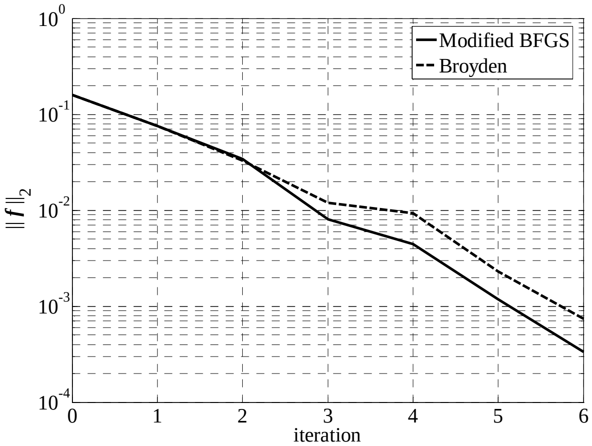

Fig. 1. $\|\boldsymbol{f}\|_2$ versus iteration for the seven-section TL capacitively loaded impedance transformer using the modified BFGS update.

TABLE I
ASM Using Broyden Rank-1 Versus Modified BFGS Rank-2 Updates for the Seven-Section Capacitively Loaded Impedance Transformer

Updating method	Iterations	$\|\boldsymbol{f}\|_2$
Broyden	6	7.4e–4
modified BFGS	6	3.3e–4

B. Six-Section H-plane Waveguide Filter

We consider the six-section H-plane waveguide filter [8]. A waveguide with a width 1.372 inches (3.485cm) is used. The six-waveguide sections are separated by seven H-plane septa, which have a finite thickness of 0.0245 inches (0.6223 mm). The design parameters are the three waveguide-section lengths L_1, L_2 and L_3 and the septa widths W_1, W_2, W_3 and W_4. A minimax objective function is employed with upper and lower design specifications as

$$\begin{aligned} &\left|S_{11}\right| \le 0.16 \text{ for } 5.4 \text{ GHz} \le \omega \le 9.0 \text{ GHz} \\ &\left|S_{11}\right| \ge 0.85 \text{ for } \omega \le 5.2 \text{ GHz} \\ &\left|S_{11}\right| \ge 0.5 \quad \text{for } \omega \ge 9.5 \text{ GHz} \end{aligned} \tag{14}$$

The fine model utilizes the time-domain full-wave

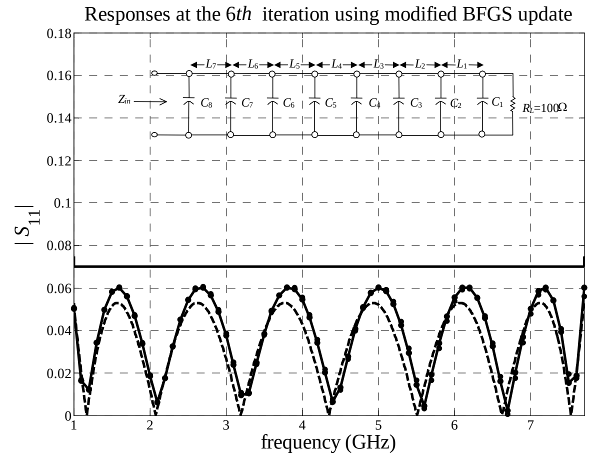

Fig. 2. Optimal coarse model response (--), optimal fine model response (–•–) and the final fine model response (•) for the seven-section transmission line capacitively loaded impedance transformer at using the modified BFGS update.

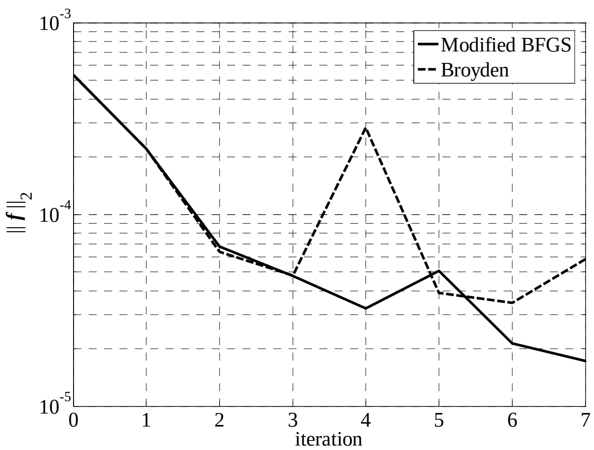

Fig. 3. $\|\boldsymbol{f}\|_2$ versus iteration for the six-section H-plane waveguide filter using the modified BFGS update.

simulator MEFiSTo [9] with a square cell $\Delta x = \Delta y = 0.6223$ mm and Johns matrix boundaries with $N_t = 8000$ time steps. We utilize 51 points in the frequency range 5.0GHz $\le \omega \le$ 10.0GHz. A coarse model with lumped inductances and dispersive transmission line sections is utilized. We simplify formulas due to Marcuvitz [10] for the inductive susceptances corresponding to the H-plane septa. They are connected to the transmission line sections through circuit theory [11].

Convergence results are given in Table II. We see that the modified BFGS update provides better convergence than the Broyden update. The reductions of $\|\boldsymbol{f}\|_2$ versus iteration using the Broyden and the modified BFGS formulas are shown in Fig. 3. The final response using the modified BFGS update is shown in Fig. 4.

IV. Conclusions

We propose a modified rank-2 BFGS updating formula for the non-symmetric rectangular case. The proposed formula is successfully examined with two examples. It provides better convergence for solving the nonlinear equations system using the aggressive SM algorithm versus the Broyden rank-1 update.

The results presented are promising. We expect that the proposed formula will enhance the convergence properties of the aggressive SM algorithm if the coarse model is badly chosen. Employing the trust region methodology to improve the algorithm convergence needs further investigation.

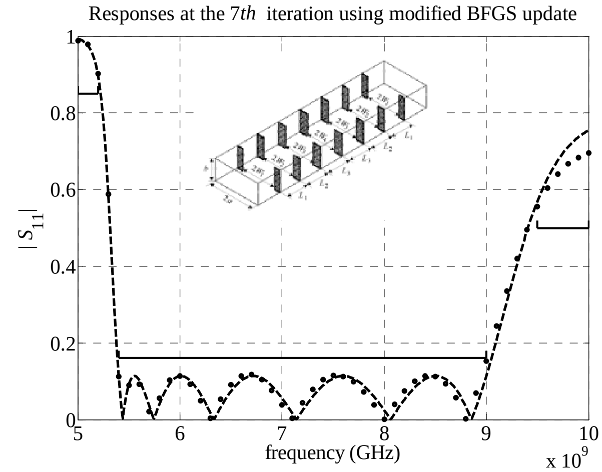

Fig. 4. Optimal coarse model response (--) and the fine model response (•) for the six-section H-plane waveguide filter at the final iteration using the modified BFGS update.

TABLE II
ASM Using Broyden Rank-1 Versus Modified BFGS Rank-2 Updates for the Six-Section H-Plane Waveguide Filter

Updating method	Iterations	$\|\boldsymbol{f}\|_2$
Broyden	7	5.8e–5
modified BFGS	7	1.7e-5

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