Q1: Three coins are tossed. What is the probability of getting (i) all heads, (ii) two heads, (iii) at least one head, (iv) at least two heads?

Sol.: Let ‘S’ be the sample space. Then $S = \{ \text{HHH}, \text{HHT}, \text{HTH}, \text{THH}, \text{HTT}, \text{THT}, \text{TTH}, \text{TTT} \}$

(i) Let ‘E1’ = Event of getting all heads, Then $E1 = \{ \text{HHH} \}$

$|E1| = 1$

$P(E1) = \frac{|E1|}{|S|} = \frac{1}{8}$

(ii) Let $E2 = \text{Event of getting '2' heads.}$ Then:

$E2 = \{ \text{HHT, HTH, THH} \}$

$|E2| = 3$

$P(E2) = \frac{3}{8}$

(iii) Let $E3 = \text{Event of getting at least one head.}$ Then:

$E3 = \{ \text{HHH, HHT, HTH, THH, HTT, THT, TTH} \}$

$|E3| = 7$

$P(E3) = \frac{7}{8}$

(iv) Let $E4 = \text{Event of getting at least one head,}$ Then:

$E4 = \{ \text{HHH, HHT, HTH, THH} \}$

$|E4| = 4$

$P(E4) = \frac{4}{8} = \frac{1}{2}$

Q2: What is the probability, that a number selected from 1, 2, 3, ..., 25, is a prime number, when each of the numbers is equally likely to be selected.

Sol.: $S = \{ 1, 2, 3, \ldots, 25 \}$  $|S| = 25$

And $E = \{ 2, 3, 5, 7, 11, 13, 17, 19, 23 \}$  $|E| = 9$

Hence $P(E) = \frac{|E|}{|S|} = \frac{9}{25}$

Q3: Two dice are thrown simultaneously. Find the probability of getting:

(i) The same number on both dice,

(ii) An even number as the sum,

(iii) A prime number as the sum,

(iv) A multiple of ‘3’ as the sum,

(v) A total of at least 0,

(vi) A doublet of even numbers,

(vii) A multiple of ‘2’ on one dice and a multiple of ‘3’ on the other dice.

Sol.: Here:

$S = \{ (1,1), (1,2), \ldots, (1,6), (2,1), (2,2), \ldots, (2,6), (3,1), (3,2), \ldots, (3,6), \ldots, (5,1), (5,2), \ldots, (5,6), (6,1), (6,2), \ldots, (6,6) \}$

$|S| = 6 \times 6 = 36$

(i) Let $E1 = \text{Event of getting same number on both side:}$

$E1 = \{ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \}$  $|E1| = 6$

$P(E1) = \frac{|E1|}{|S|} = \frac{6}{36} = \frac{1}{6}$

(ii) Let $E2 = \text{Event of getting an even number as the sum.}$

$E2 = \{ (1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6) \}$

$|E2| = 18$  hence $P(E2) = \frac{|E2|}{|S|} = \frac{18}{36} = \frac{1}{2}$

(iii) Let $E3 = \text{Event of getting a prime number as the sum.}$

$E3 = \{ (1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5) \}$
(iv) Let $E_4 = \text{Event of getting a multiple of '3' as the sum.}$

$E_4 = \{(1,2), (1.5), (2,1), (2,4), (3,3), (3,6), (4,2), (4,5), (5,1), (5,4), (6,3), (6,6),\}$

$|E_4| = 12$

$P(E_4) = \frac{|E_4|}{|S|} = \frac{12}{36} = \frac{1}{3}$

(v) Let $E_5 = \text{Event of getting a total of at least 10.}$

$E_5 = \{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6),\}$

$|E_5| = 6$

$P(E_5) = \frac{|E_5|}{|S|} = \frac{6}{36} = \frac{1}{6}$

(vi) Let $E_6 = \text{Event of getting a doublet of even numbers.}$

$E_6 = \{(2,2), (4,4), (6,6),\}$

$|E_6| = 3$

$P(E_6) = \frac{|E_6|}{|S|} = \frac{3}{36} = \frac{1}{12}$

(vii) Let $E_7 = \text{Even of getting a multiple of '2' on one dice and a multiple of '3' on the other dice.}$

$E_7 = \{(2,3), (2,6), (4,3), (4,6), (6,3), (3,2), (3,4), (3,6), (6,2), (6,4)\}$

$|E_7| = 11$

$P(E_7) = \frac{|E_7|}{|S|} = \frac{11}{36}$

Q4.: What is the probability, that a leap year selected at random will contain 53 Sundays?

Sol.: A leap year has 366 days, therefore 52 weeks i.e. 52 Sunday and 2 days.

The remaining 2 days may be any of the following:

(i) Sunday and Monday
(ii) Monday and Tuesday
(iii) Tuesday and Wednesday
(iv) Wednesday and Thursday
(v) Thursday and Friday
(vi) Friday and Saturday
(vii) Saturday and Sunday

For having 53 Sundays in a year, one of the remaining 2 days must be a Sunday.

$|S| = 7; \quad |E| = 2$

$P(E) = \frac{|E|}{|S|} = \frac{2}{7}$

Q5.: Two cards are drawn at random. Find the probability that both the cards are of red colour or they are queen.

Sol.: Let $S$ = Sample – space.

$A = \text{The event that the two cards drawn are red.}$

$B = \text{The event that the two cards drawn are queen.}$

$A \cap B = \text{The event that the two cards drawn are queen of red colour.}$

$|S| = C(52,2), \quad |A| = C(26,2), \quad |B| = C(4,2)$

$n(A \cap B) = C(2,2)$

$P(A) = \frac{|A|}{|S|} = C(26,2) / C(52,2), \quad P(B) = \frac{|B|}{|S|} = C(4,2) / C(52,2)$

$P(A \cap B) = \frac{|A \cap B|}{|S|} = C(2,2) / C(52,2)$

$P(A \cup B) = ?$

We have $P(A \cup B) = P(A) + P(B) - P(A \cap B) = C(26,2) / C(52,2) + C(4,2) / C(52,2) - C(2,2) / C(52,2) = 55/221$